$$
\begin{aligned}
a_1 &= \cos\beta\cos\gamma & a_3 &= \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma \\
b_1 &= -\cos\beta\sin\gamma & b_3 &= \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma \\
c_1 &= \sin\beta & c_3 &= \cos\alpha\cos\beta
\end{aligned} \tag{11}
$$

Portions of the potential which are independent of angular orientation have been omitted from (10), which is seen to be a form of MacCullagh's expression for the potential of a distant attracting mass.

It is hardly necessary to emphasize that the result enunciated in Ref. 1 is correctly scrutinized by applying the more succinct principle (7) without approximation or special assumptions, as indicated previously. Readers who might still cling to the excessively differentiated system (1–3) should recognize that (4) demonstrates an invariance of the differential system with respect to displacement of origin of angular coordinates; this provides an alternative interpretation of the result (5) or (7).

The angle variables employed in the present discussion are seen to possess the same advantages of convenience secured in a different manner by Jeffreys.[2]

For more complete discussions of special integrals of energy, of permanent configurations and surfaces of zero velocity, of effects of gyroscopic terms, and of other concepts utilized in the preceding discussion, the reader should consult standard treatises such as Refs. 3 and 4.

### References

[1] Michelson, I., "Equilibrium orientations of gravity-gradient satellites," AIAA J. 1, 493 (1963).

[2] Jeffreys, H., "The moon's principal librations in rectangular coordinates," Monthly Notices Roy. Astronom. Soc. **115,** 189 (1955).

[3] Brouwer, D. and Clemence, G. M., *Methods of Celestial Mechanics* (Academic Press, Inc., New York, 1961), Chap. X.

[4] Moulton, F. R., *An Introduction to Celestial Mechanics* (Macmillan Company, New York, 1914), revised ed., Chap. VII.

## Comment on "Coning Effects Caused by Separation of Spin-Stabilized Stages"

R. O. Wilke*

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In a previous note,[1] Dwork discussed a separation effect that produces instantaneous variation in coning angle upon separation of spin-stabilized stages. The effect is purely mechanical and owes its origin to simple conservation of the moment of momentum vector. However, somewhere between the desk pen and printed copy an error arises which could result in serious consequences should the equations be applied directly. A correction of the equations will also result in a clearer physical picture.

Following Dwork, we start with the magnitude of the moment of momentum vectors about the longitudinal and transverse axes prior to separation of the two bodies:

$$
\left.\begin{aligned}
M_L &= (I_{L_1} + I_{L_2})\omega_L \\
M_T &= (I_{T_1} + I_{T_2})\omega_T + \mu l^2 \omega_T
\end{aligned}\right\} \quad \mu = \frac{m_1 m_2}{(m_1 + m_2)}
$$

The term $\mu l^2$ can easily be shown to represent a contribution to transverse moment of inertia through axis transformation and should appear in the transverse equation as just shown and not in the longitudinal.

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The rest of the discussion follows in order, but the following observation might be made. The coning angle just prior to separation is defined as

$$
\tan\theta_{\text{sep}} = \frac{M_T}{M_L} = \frac{(I_{T_1} + I_{T_2} + \mu l^2)\omega_T}{(I_{L_1} + I_{L_2})\omega_L}
$$

and after separation it becomes (for the first mass)

$$
\tan\theta_1 = \frac{M_{T_1}}{M_{L_1}} = \frac{I_{T_1}}{I_{L_1}} \tan\theta_{\text{sep}} \frac{(I_{L_1} + I_{L_2})}{(I_{T_1} + I_{T_2} + \mu l^2)}
$$

Therefore,

$$
\tan\theta_1 = \left(\frac{I_T}{I_L}\right)_{\text{payload}} \tan\theta_{\text{sep}} \left(\frac{I_L}{I_T}\right)_{\text{combined}}
$$

The transverse moment of inertia term with the transformation quantity $\mu l^2$ merely means the transverse moment of inertia about the common center of gravity. Since this quantity is usually measured for each vehicle and combination of vehicles, it might be more convenient to refer to the foregoing in terms of moment of inertia ratios before and after separation. This ratio comparison gives a better physical idea of the losses or gains to be expected instaneously even with a "perfect" separation system:

$$
\frac{\tan\theta_1}{\tan\theta_{\text{sep}}} = \frac{(I_L/I_T)_{\text{combined}}}{(I_L/I_T)_{\text{payload}}}
$$

### Reference

[1] Dwork, M., "Coning effects caused by separation of spin-stabilized stages," AIAA J. 1, 2639–2640 (1963).

## Reply by Author to R. O. Wilke

M. J. Dwork*

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The author wishes to thank R. O. Wilke. The comments made are correct, errors do exist in the subscripts. Moreover, the relations he derives are simpler in form to use.

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## Comments on "Integral Approach to an Approximate Analysis of Thrust Vector Control by Secondary Injection"

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It is instructive to compare the results of the present paper[1] with other work,[2, 3] which also used an integral approach to the problem of thrust vector control by secondary injection. The assumptions of Ref. 2 and 3 were markedly different than those of the Ref. 1. Specificially, the mathematical model was based on one-dimensional, inviscid, isentropic

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